

Maths Boot Camp: Session 4 Exercises

Counting, Probabilities, Random Variables

Department of Government, London School of Economics & Political Science

This sheet collects further practice on Session 4's three blocks: counting; probability; and random variables. Worked solutions are available on request from the author (m.antonetti@lse.ac.uk).

Topic 1: Counting

Example 1: Fundamental Principle of Counting

Problem: You have 3 shirts and 4 pairs of pants. How many different outfits can you wear?

Example 2: Permutations Without Replacement

Problem: How many ways can you arrange 2 books out of 5 on a shelf?

Example 3: Combinations

Problem: How many ways can you choose 2 fruits from a basket of 5 different fruits?

Example 4: License Plates

Problem: A license plate consists of 3 letters followed by 3 digits. How many different license plates can be formed?

Topic 2: Probability

Example 1: Conditional Probability

Problem: In a class, 30% of students have a dog, and 20% have a cat. The probability of having both is 10%. What is the probability a student has a cat, given that they have a dog?

Example 2: Law of Total Probability

Problem: A factory has two machines. Machine 1 produces 70% of items with a 1% defect rate; Machine 2 produces 30% with a 2% defect rate. What is the probability a randomly selected item is defective?

Example 3: Bayes' Theorem

Problem: A doctor knows that 1% of patients have a particular disease. If a patient has the disease, the probability of a positive test is 90%. If a patient does not have the disease, the probability of a positive test is 5%. What is the probability that a patient has the disease, given they tested positive?

Topic 3: Random Variables

Example 1: Expected Value

Problem: Find the expected value of rolling a fair six-sided die.

Example 2: Variance and Standard Deviation

Problem: For $X \sim \text{Binomial}(n = 5, p = 0.5)$, find the variance and standard deviation.

Example 3: Levels of Measurement

Problem: Classify the following as nominal, ordinal, interval, or ratio: (a) Zip code; (b) Daily temperature in Fahrenheit; (c) Educational level (High School, Bachelor's, Master's, PhD); (d) Number of pets owned.