

Maths Boot Camp: Session 3 Exercises

Derivatives, Optimisation, Integrals

Department of Government, London School of Economics & Political Science

This sheet collects further practice on Session 3's three blocks: derivatives; maximisation and minimisation; and integrals. Worked solutions are available on request from the author (m.antonetti@lse.ac.uk).

Topic 1: Derivatives

Example 1: Basic Derivative

Problem: Find the derivative of $f(x) = 3x^3 - 5x + 2$.

Example 2: Local Extrema

Problem: Determine the local maximum and minimum points of $f(x) = x^3 - 3x^2 + 4$.

Topic 2: Maximisation and Minimisation

Example 1: Maximising Profit

Problem: A company's profit $P(x)$, in thousands of dollars, is $P(x) = -2x^2 + 12x - 20$, where x is the number of units produced (in hundreds). How many units should the company produce to maximise profit?

Example 2: Maximum on an Interval

Problem: Find the maximum value of $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$.

Example 3: Minimising Cost

Problem: A factory's cost function $C(x)$, in thousands of dollars, is $C(x) = 3x^2 - 12x + 15$, where x is the number of units produced (in hundreds). How many units minimise cost?

Example 4: Minimum on an Interval

Problem: Find the minimum value of $g(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 4]$.

Topic 3: Integrals

Example 1: Basic Indefinite Integral

Problem: Compute $\int (2x + 3) dx$.

Example 2: Definite Integral

Problem: Evaluate $\int_1^3 4x^3 dx$.

Example 3: Area Under a Curve

Problem: Find the area under $y = x^2$ from $x = 0$ to $x = 2$.