

Maths Boot Camp: Session 2 Exercises

Vectors, Matrices, Limits

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This sheet collects further practice on Session 2's three blocks: vectors; matrices; and neighbourhoods, continuity and limits. Worked solutions are available on request from the author (m.antonetti@lse.ac.uk).

Topic 1: Vectors

Example 1: Vector Addition

Problem: Given $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$, find $\mathbf{a} + \mathbf{b}$.

Example 2: Inner Product

Problem: Let $\mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$. Compute $\mathbf{u} \cdot \mathbf{v}$.

Example 3: Euclidean Distance

Problem: Find the Euclidean distance between the points $P = (2, -1)$ and $Q = (5, 3)$.

Topic 2: Matrices

Example 1: Matrix Multiplication

Problem: Compute the product of $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$.

Example 2: Properties of Matrix Multiplication

Problem: Given $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ 2 & 0 \end{pmatrix}$, show that $AB \neq BA$.

Example 3: Matrix Transpose

Problem: Find the transpose of $A = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 4 & 5 \end{pmatrix}$.

Topic 3: Neighbourhoods and Limits

Example 1: Evaluating a Limit

Problem: Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$.

Example 2: Continuity of a Function

Problem: Determine if $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$.

Example 3: A Limit at Infinity

Problem: Evaluate $\lim_{x \rightarrow \infty} \frac{3x + 2}{x}$.