

Maths Boot Camp — Session 1

The Basics, Functions, and Logarithms

Formula & Definition Cheat Sheet — keep this to hand during class

1. Number Systems

- **N Natural:** $\{1, 2, 3, \dots\}$
- **W Whole:** $\{0, 1, 2, 3, \dots\}$
- **Z Integers:** $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- **Q Rational:** ratios of integers, e.g. $\frac{3}{5}$
- **R Real:** rationals + irrationals, e.g. $\pi, \sqrt{2}$
- **C Complex:** $z = a + bi$

Each is nested inside the next: $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

2. Intervals

$\equiv$ “is equivalent to” $\in$ “belongs to” : “such that”

- **Open:** $(a, b) \equiv \{x \in \mathbb{R} : a < x < b\}$ — excludes both ends
- **Closed:** $[a, b] \equiv \{x \in \mathbb{R} : a \leq x \leq b\}$ — includes both ends
- **Half-open:** $(a, b] \equiv \{x \in \mathbb{R} : a < x \leq b\}$

3. Sets

- **Union** $A \cup B$: everything in A or B
- **Intersection** $A \cap B$: only what's in *both*
- **Difference** $A - B$: elements of A *not* in B — stays inside A , never adds anything from B
- **Symmetric difference** $A \triangle B = (A - B) \cup (B - A)$: everything in exactly one of the two sets — *not the same as* $A - B$
- **Complement** A^c : everything in the universal set not in A
- **Proper subset** $A \subset B$: every element of A is in B , and $A \neq B$
- **Improper subset** $A \subseteq B$: allows $A = B$
- **Empty set:** $\emptyset$ or $\{\}$

Common mix-up: $A - B \neq \{\text{leftovers of both sides}\}$ — that combined set is $A \triangle B$, a different operation.

Open / closed / bounded sets:

- **Boundary point:** touches both the set and its complement
- **Interior point:** safely inside, away from any edge
- **Bounded:** doesn't run off to infinity
- **Unbounded:** extends infinitely in some direction
- **Compact:** closed *and* bounded together

4. Common Operators — one equation, three questions

All three of power, root, and log are just different unknowns in

$$a^b = c$$

- **Power** $a^b = c$: given a, b , find c
- **Root** $\sqrt[b]{c} = a$: given b, c , find a — c is exactly the power's result, and also what sits *under* the root
- **Logarithm** $\log_a c = b$: given a, c , find b
- **Summation** $\sum_{i=1}^n a_i$: adds a sequence, $a_1 + a_2 + \dots + a_n$
- **Factorial** $n! = n \times (n-1) \times \dots \times 1$: product of all positive integers up to n

5. Properties of Powers

Rule	Example
$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^4 = 2^7$
$\frac{a^m}{a^n} = a^{m-n}$	$5^6 / 5^2 = 5^4$
$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^8$
$(ab)^n = a^n b^n$	$(2x)^3 = 2^3 x^3$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(x/2)^3 = x^3 / 8$
$a^0 = 1$	$7^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = 1/8$
$a^{1/n} = \sqrt[n]{a}$	$16^{1/2} = 4$
$a^{m/n} = \left(\sqrt[n]{a}\right)^m$	$8^{2/3} = 4$

Watch out: $a^m + a^n$ cannot be simplified by adding exponents — and $a^3 \cdot a^3 = a^6$ (add), not a^9 (that would be $(a^3)^3$).

6. Properties of Logarithms

$\log_a c = b$ means $a^b = c$ ($a > 0, a \neq 1, c > 0$)

Rule	Example
$\log_a(bc) = \log_a b + \log_a c$	$\log(1000) = \log 10 + \log 100 = 3$
$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$	$\log(100/10) = 2 - 1 = 1$
$\log_a(b^c) = c \log_a b$	$\log(100^2) = 2 \log 100 = 4$
$\log_a 1 = 0$	$\log_5(1) = 0$
$\log_a a = 1$	$\log_7(7) = 1$
$\log_a(a^x) = x$	$\log_2(2^5) = 5$
$a^{\log_a x} = x$	$10^{\log(7)} = 7$
$\log_a b = \frac{\log_c b}{\log_c a}$	$\log_2 8 = \ln 8 / \ln 2 = 3$

Watch out: $\log(x+y) \neq \log x + \log y$ — the product/quotient rules apply to multiplication/division *inside* the log, not addition/subtraction.

7. Order of Operations (PEMDAS)

Parentheses $\rightarrow$ Exponents $\rightarrow$ Multiplication/Division (left to right) $\rightarrow$ Addition/Subtraction (left to right)

8. Solving Equations

Linear: undo $+/-$ first, then undo $\times/\div$.

E.g. $5x - 7 = 18 \Rightarrow 5x = 25 \Rightarrow x = 5$.

Factorising a quadratic: find two numbers that multiply to c and sum to b in $x^2 + bx + c$.

E.g. $x^2 + 5x + 6$: numbers are $2, 3 \Rightarrow (x + 2)(x + 3) = 0 \Rightarrow x = -2, -3$.

Quadratic formula — for any $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is the **discriminant**; its sign gives the number of real roots. The $\pm$ gives (generically) two solutions.

Worked example: $2x^2 - 4x - 6 = 0 \Rightarrow a=2, b=-4, c=-6$. Discriminant $= 16 + 48 = 64$. $x = \frac{4 \pm 8}{4} \Rightarrow x = 3$ or $x = -1$.

9. Inequalities

Compare with $>, \geq, <, \leq$. Solve the same way as equations, **except**:

Dividing (or multiplying) both sides by a **negative number** reverses the inequality sign.

E.g. $-3x \geq 9 \Rightarrow \frac{-3x}{-3} \leq \frac{9}{-3} \Rightarrow x \leq -3$.

10. Functions

A **function** $f(x) : A \rightarrow B$ maps every input in domain A to exactly *one* output in co-domain B . (Different x 's can share a y ; one x with two y 's is a *relation*, not a function.)

Types:

- **Monomial:** $f(x) = ax^k$, e.g. $y = x^2$
- **Polynomial:** sum of monomials, e.g. $y = 3x + 5$
- **Rational:** ratio of two polynomials, e.g. $y = \frac{x^2 + 1}{2x^2 - 2x + 1}$
- **Exponential:** e.g. $y = 2^x$
- **Trigonometric:** e.g. $y = \cos(x)$
- **Linear:** degree 1, $y = mx + b$ **Nonlinear:** anything else

Root of a function: a value x such that $f(x) = 0$ — finding roots of f = solving $f(x) = 0$ (same steps as §8, e.g. the quadratic formula).

Composite functions: apply one function to the output of another: $f[g(x)]$. Work from the inside out. Order matters: $h[f(x)] \neq f[h(x)]$ in general.

E.g. $h(x) = x^2$, $f(x) = x - 5$: $h(f(2)) = h(-3) = 9$.

E.g. $f(x) = 2x$, $g(x) = x + 4$: $f(g(x)) = 2(x + 4) = 2x + 8$.